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Homogeneous fluxes, branes and a maximally supersymmetric solution of M-theory

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Abstract: We find M-theory solutions with homogeneous fluxes for which the spacetime is a lorentzian symmetric space. We show that generic solutions preserve sixteen supersymmetries and that there are two special points in their moduli space of parameters which preserve all thirty-two supersymmetries. We calculate the symmetry superalgebra of all these solutions. We then construct various M-theory and string theory branes with homogeneous fluxes and we also find new homogeneous flux-brane solutions.

Keywords: M-Theory, p-branes, Differential and Algebraic Geometry.

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Contents

1. Introduction

Solutions of supergravity theory that preserve all spacetime supersymmetry have played a central rôle in the understanding of the properties of superstrings and M-theory. The applications range from the quantisation of superstrings in ten-dimensional Minkowski spacetime to the formulation of the AdS/CFT correspondence. Most of our understanding is based on the large number of unbroken supersymmetries that such solutions exhibit. In M-theory, it is well known that backgrounds that preserve all thirty-two supersymmetries include eleven-dimensional Minkowski space, $AdS_4 \times S^7$ and $AdS_7 \times S^4$. Less well known is that in addition to the above three spaces there is another solution of $D = 11$ supergravity theory which preserves all supersymmetries. This solution was discovered by Kowalski-Glikman [1] and in what follows we shall refer to it as the KG space or solution.

In this paper, we shall investigate solutions of supergravity theories in ten and eleven dimensions that have covariantly constant form field-strength fluxes. In this paper we will call these fluxes *homogeneous*, since the spacetimes we will be concerned with will be symmetric spaces, and for a symmetric space these two concepts agree. We therefore define a homogeneous p-brane (or Hp-*brane*) to be a solution with $\mathrm{ISO}(p,1)$ Poincaré invariance and with a U-duality representative which has homogeneous fluxes. Such solutions do not necessarily carry usual p-brane charges but as we shall see in some cases describe the asymptotic geometry of p-branes in the presence of a homogeneous flux. We shall also find that our H-branes are closely related to flux-branes (or F-branes). In fact, we shall show that if instead of reducing a flat spacetime [2, 3], we reduce a certain H-brane spacetime, we obtain an F-brane in the presence of additional fluxes.

We shall focus on the H-branes that are constructed from a large family of eleven-dimensional lorentzian symmetric spacetimes found by Cahen and Wallach [4]. We call them Cahen-Wallach (CW) spaces.[1] In particular we shall show that generic CW spaces equipped with a homogeneous four-form null flux are solutions of eleven-dimensional supergravity preserving one half of the supersymmetry. The resulting solution is an M-theory pp-wave in homogeneous four-form flux, or Hpp-wave. Our solutions include a spacetime with homogeneous fluxes found in [5].

We shall then investigate the moduli space of these solutions and we shall find that there are two special points. One is eleven-dimensional Minkowski spacetime and the other is the KG spacetime. At both of these points there is supersymmetry enhancement and the solutions preserve all thirty-two supersymmetries of M-theory.

We shall find the Killing vectors and Killing spinors, and compute the symmetry superalgebra of Hpp-waves. The Killing vectors and Killing spinors of the KG solution were computed explicitly in [6]. Anticipating the importance that the KG solution may have in the context of M-theory, we shall compute its symmetry superalgebra. We shall find that the dimension of the bosonic part of the algebra of the KG solution is 38 which is the same as that of the $AdS_4 \times S^7$ and $AdS_7 \times S^4$ solutions of M-theory. However unlike the latter two cases where the isometry group is semisimple, the isometry group of KG solution is not. The symmetry superalgebra of the rest of the Hpp-waves will also be given.

We shall investigate the reduction of the M-theory Hpp-wave that we have found and in particular that of the KG spacetime to IIA string theory. The reduced solution is a IIA H0-brane. It is known that in some cases after Kaluza-Klein reduction, the reduced solution preserves less supersymmetry than the original one, see for example [7, 8, 9]. It has been argued in [10] for the special case of T-duality on toric hyperkähler manifold, but applies more generally, that a necessary condition for preserving supersymmetry in supergravity theory is that the Lie derivative of

[1] No confusion should arise with the unrelated notion of a CW complex, familiar to topologists.

the Killing spinors along the direction of compactification vanishes. This condition does not depend on the choice of coordinates and the choice of frame of spacetime. More generally, the number of supersymmetries preserved in supergravity theory after reducing a supersymmetric solution is equal to the number of Killing spinors which have vanishing Lie derivatives along the direction of compactification. We shall see that the above condition on the Killing spinors is equivalent to requiring that the supercharges associated with the reduced solution are those of the original supersymmetric solution that commute with the generator of translations along the compact direction. In particular we find that for the generic Hpp-wave the radius of the compact direction can be chosen such that the solution preserves one half of the supersymmetry in eleven dimensions. However, the Lie derivative along the compact direction of the Killing spinors of an Hpp-wave does not vanish. Therefore, the associated H0-brane does not preserve any supersymmetry in the context of IIA supergravity. This is also reminiscent of the backgrounds with supersymmetry without supersymmetry of [11]. For the Hpp-wave associated with the KG spacetime, the radius of the compact direction can be chosen such that the solution preserves thirty-two or sixteen supersymmetries in eleven dimensions. However again, the Lie derivative of the Killing spinors does not vanish along the compact direction and so the associated H0-brane does not preserve any supersymmetry in IIA supergravity. Using U-duality one can construct Hp-branes in IIA (p even) and IIB (p odd) supergravities.

We shall also consider the superposition of our H-brane solutions with the standard M-branes. We shall in fact focus on the superposition of an M-theory pp-wave [12] with an Hpp-wave. We shall find that upon reduction to IIA supergravity, the solution has the interpretation of D0-branes in the presence homogeneous fluxes, or HD0-branes. Using U-duality, one can then construct HDp- and HNSp-branes in type-II theories.

Our H-brane solutions can also be used to construct a class of flux-branes with additional homogeneous fluxes (HF-*branes*, for short). For this, we shall perform a reduction in a class of our M-theory Hpp-wave solutions. The resulting IIA configuration has a magnetic two-form flux characteristic of a F7-brane, in addition to the fluxes which are reductions of the homogeneous four-form field strength.

This paper has been organised as follows. In section 2, we summarise the construction of CW spaces and investigate their geometry. In section 3 we give the Hpp-wave solutions of eleven-dimensional supergravity. In section 4 we compute the Killing spinors for the Hpp-waves and those of KG spacetime. In section 5 we examine the isometries of the Hpp-waves. In section 6 we give the symmetry superalgebra of Hpp-waves and in particular that of KG spacetime. In section 7 we use U-duality to construct many H-brane solutions. In section 8 we use the spinorial Lie derivative to investigate the number of supersymmetries preserved by a solution after a Kaluza-Klein reduction. We then apply our results in the context of H-branes. In section 9

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we find HD- and HNS- brane solutions. In section 10 we find HF-brane solutions. Finally, in section 11, we discuss the moduli space of maximally supersymmetric solutions of eleven-dimensional supergravity.

2. Cahen-Wallach lorentzian symmetric spaces

In this section we present the construction due to Cahen and Wallach [4] of a family of indecomposable lorentzian symmetric spaces. This family exists for any dimension ≥ 3, but we will consider only the eleven-dimensional case — the general case following straightforwardly from this one. Let $x^{\pm}$ be light-cone coordinates, $x^i; i = 1, \ldots, 9$ be coordinates in $\mathbb{R}^9$ and $A = (A_{ij})$ be a real symmetric matrix. The CW metric is

$$ds^2 = 2dx^+dx^- + \sum_{i,j} A_{ij}x^ix^j(dx^-)^2 + \sum_i dx^idx^i \,. \tag{2.1}$$

It is not hard to show that the resulting space is indecomposable (that is, it is not locally isometric to a product) if and only if A is non degenerate. On the other hand if A is degenerate then the metric (2.1) decomposes to a $(11-k)$-dimensional indecomposable CW space and the standard metric on $\mathbb{R}^k$, where k is the dimension of the radical of A.

In order to make the dependence of the CW metric on A explicit, we will let M_A denote the space with CW metric corresponding to the matrix A. Some of the CW spaces associated with different matrices A are isometric. To be precise, two CW spaces M_{A_1} and M_{A_2} are (locally) isometric if and only if $A_1 = c^2OA_2O^{-1}$, where c is a non-vanishing (real) constant and O is an orthogonal transformation. Indeed, the diffeomorphism is given explicitly by rotating the x^i by O and then rescaling $x^{\pm} \mapsto c^{\pm 1}x^{\pm}$.

The moduli space of eleven-dimensional CW metrics is that of the unordered eigenvalues of A up to a positive scale. This space is diffeomorphic to S^8/Σ_9 where Σ_9 is the permutation group of nine objects acting in the obvious way on the homogeneous coordinates of S^8, that is, on the eigenvalues $(\lambda_1, \ldots, \lambda_9)$ of the matrix A. The moduli space of indecomposable CW metrics is

$$\frac{(S^8\backslash\Delta)}{\Sigma_9}\,, \qquad \text{where } \Delta = \{(\lambda_1, \ldots, \lambda_9) \in S^8 \,|\, \lambda_1\lambda_2\cdots\lambda_9 = 0\}\,,$$

which is non compact. Adding the decomposable CW metrics compactifies the moduli space to S^8/Σ_9. This provides a simple example of a phenomenon that has been observed in the compactification of many other moduli spaces.

To show that the metric (2.1) is complete, to find the isometries and to understand the global structure of M_A, we shall construct M_A as a symmetric space. To this end, let V be a real nine-dimensional vector space and let V^* be the dual space. We may identify V and V^* with $\mathbb{R}^9$ by choosing a basis $\{e_i\}$ for V and canonical dual basis $\{e_i^*\}$ for V^*. We define a euclidean inner product on V by declaring the e_i to be an orthonormal basis. Let Z be a one-dimensional real vector space and let Z^* be its dual space. We identify them with $\mathbb{R}$ by choosing canonically dual bases e_+ for Z and e_- for Z^*. Finally let A be a real symmetric bilinear form on V: $A(e_i, e_j) = A_{ij} = A_{ji}$. Then the twenty-dimensional vector space $V \oplus V^* \oplus Z \oplus Z^*$ can be made into a solvable Lie algebra with the following non-zero Lie brackets:

$$[e_-, e_i] = e_i^*\,, \qquad [e_-, e_i^*] = \sum_j A_{ij} e_j\,, \qquad [e_i^*, e_j] = A_{ij} e_+\,. \tag{2.2}$$

The brackets satisfy the Jacobi identity by virtue of A being symmetric. We call the resulting Lie algebra $\mathfrak{g}_A$, since it depends on A. Notice that the second derived ideal $\mathfrak{g}_A''$ is central, whence $\mathfrak{g}_A$ is solvable. The Lie algebra (2.2) is isomorphic to a Heisenberg algebra generated by $\{e_i, e_i^*, e_+\}$ and equipped with an outer automorphism e_- which rotates positions $\{e_i\}$ and momenta $\{e_i^*\}$. Systems with such symmetry include the harmonic oscillator, the Planck "constants" being given by the eigenvalues of A.

Let $\mathfrak{k}_A$ denote the abelian Lie subalgebra spanned by the $\{e_i^*\}$ and let $\mathfrak{p}_A$ denote the complementary subspace, spanned by $\{e_i, e_+, e_-\}$. It is clear from the Lie brackets (2.2) that

$$[\mathfrak{k}_A, \mathfrak{p}_A] \subset \mathfrak{p}_A\,, \qquad \text{and} \qquad [\mathfrak{p}_A, \mathfrak{p}_A] \subset \mathfrak{k}_A\,,$$

whence $\mathfrak{g}_A = \mathfrak{k}_A \oplus \mathfrak{p}_A$ is a symmetric split. Furthermore, the action of $\mathfrak{k}_A$ on $\mathfrak{p}_A$ preserves the symmetric bilinear form

$$B(e_i, e_j) = \delta_{ij}\,, \qquad \text{and} \qquad B(e_+, e_-) = 1\,. \tag{2.3}$$

Let G_A denote the simply-connected Lie group with Lie algebra $\mathfrak{g}_A$ and let K_A denote the Lie subgroup corresponding to the Lie subalgebra $\mathfrak{k}_A$. Then the space $M_A = G_A/K_A$ of right K_A-cosets in G_A inherits a lorentzian metric from the K_A-invariant bilinear form B on $\mathfrak{p}_A$. In this way, M_A becomes a lorentzian symmetric space with solvable transvection group G_A.

To express the metric on M_A as in (2.1), we choose a representative $\sigma : M_A \to G_A$ of the coset as

$$\sigma(x^+, x^-, x^i) = \exp(x^+ e_+) \exp(x^- e_-) \exp\left(\sum_i x^i e_i\right)\,, \tag{2.4}$$

where $\{x^+, x^-, x^i\}$ are local coordinates as in (2.1). (Normally this can only be done locally, but in this case, since M_A is contractible, we can choose a global representative.) The pull-back to M_A of the left-invariant Maurer-Cartan form on G_A can be written as

$$\sigma^{-1} d\sigma = \omega + \theta\,,$$

where the canonical $\mathfrak{k}_A$-connection ω of the coset and the $\mathfrak{p}_A$-valued soldering form θ (whose components give rise to a frame) are given by

$$\omega = dx^- \sum_i x^i e_i^*$$

and

$$\theta = dx^- e_- + \sum_i dx^i e_i + \left(dx^+ + \frac{1}{2} \sum_{i,j} A_{ij} x^i x^j dx^- \right) e_+ ,$$

respectively. In the above coordinates, the G_A-invariant metric $B(\theta, \theta)$ on M_A coincides with (2.1).

For later use, the Riemann curvature and Ricci tensors of M_A have the following non-zero components

$$R_{-i-j} = -A_{ij} , \qquad \text{and} \qquad R_{--} = -\operatorname{tr} A . \tag{2.5}$$

In particular, the scalar curvature vanishes.

The holonomy of M_A is isomorphic to $K_A \cong \mathbb{R}^9$ and the representation is induced by the adjoint action of K_A on $\mathfrak{p}_A$. It is not hard to show that the K_A-invariant subspace of $\bigwedge \mathfrak{p}_A$ is spanned by the constants together with polyvectors of the form $e_+ \wedge \theta$, where $\theta \in \bigwedge V$. This means that the corresponding parallel forms on M_A are the constants together with $dx^- \wedge \Theta$ where Θ is any constant-coefficient form

$$\Theta = \sum_{1 \leq i_1 < i_2 < \cdots < i_p \leq 9} c_{i_1 i_2 \cdots i_p} dx^{i_1} \wedge dx^{i_2} \wedge \cdots \wedge dx^{i_p} .$$

3. Hpp-waves from CW spaces

The CW spaces M_A described in the previous section can be used to construct Hpp-wave solutions of eleven-dimensional supergravity. The bosonic fields of eleven-dimensional supergravity are the metric g and the four-form field-strength F. Since we want the Hpp-wave to have the symmetries of the CW spaces, it is natural to demand that F be homogeneous, so that we shall choose F to be a parallel form on the CW space. In particular, we shall show that

$$\begin{aligned} ds^2 &= 2dx^+ dx^- + \sum_{i,j} A_{ij} x^i x^j (dx^-)^2 + \sum_i dx^i dx^i , \\ F &= dx^- \wedge \Theta , \end{aligned} \tag{3.1}$$

where Θ is a 3-form on $\mathbb{R}^9$ with constant coefficients, is a supersymmetric solution of eleven-dimensional supergravity provided that

$$\operatorname{tr} A = -\frac{1}{2} \|\Theta\|^2 = -\frac{1}{12} \Theta_{ijk} \Theta^{ijk} . \tag{3.2}$$

In the conventions of [13], the field equations of eleven-dimensional supergravity [14] are

$$
\begin{aligned}
d\star F &= \frac{1}{2} F\wedge F\,,\\
R_{MN} &= \frac{1}{12}\left(F_{MPQR}F_N{}^{PQR} - \frac{1}{12} g_{MN} F_{PQRS}F^{PQRS}\right).
\end{aligned} \tag{3.3}
$$

For later use, we also give the Killing spinor equations of eleven-dimensional supergravity. Let $\{\Gamma_a\}$ be a basis in the Clifford algebra

$$
\Gamma_a\Gamma_b + \Gamma_b\Gamma_a = 2\eta_{ab}\mathbb{1}\,,
$$

where η_{ab} is the "mostly plus" frame metric. A convenient basis for investigating Killing spinors for the solution (3.7) is one for which $\eta_{+-} = 1$ and $\eta_{ij} = \delta_{ij}$. The Killing spinors ε satisfy the equation

$$
\nabla_M \varepsilon = \Omega_M \varepsilon\,, \tag{3.4}
$$

where

$$
\Omega_M = \frac{1}{288} F_{PQRS}\left(\Gamma^{PQRS}{}_M + 8\Gamma^{PQR}\delta^S_M\right), \tag{3.5}
$$

and the spin connection ∇ is

$$
\nabla_M = \partial_M + \frac{1}{4}\omega_M{}^{ab}\Gamma_{ab}\,. \tag{3.6}
$$

To see that (3.1) satisfies the field equations of eleven-dimensional supergravity theory, we remark that F is parallel, hence it is both closed and coclosed. Moreover it obeys $F\wedge F = 0$. Therefore F obeys its equation of motion. Since F is null — that is, $F_{PQRS}F^{PQRS} = 0$ — the Einstein equations simplify to

$$
R_{MN} = \frac{1}{12} F_{MPQR}F_N{}^{PQR}\,.
$$

The only non-zero component of the Ricci tensor is $R_{--} = -\operatorname{tr} A$, and similarly the only non-zero component of energy-momentum tensor $T_{MN} := F_{MPQR}F_N{}^{PQR}$ is $T_{--} = 6\|\Theta\|^2$, whence we see that in order to obtain a bosonic solution, all we need to impose is the condition (3.2).

It is clear from our construction that the solution (3.1) is invariant under the action of the Lie algebra $\mathfrak{g}_A$ of the transvection group and that it has the interpretation of an Hpp-wave according to the definition given in the introduction. The Hpp-wave (3.1) may be invariant under the action of a larger group. A detailed investigation of the symmetries will be presented in sections 5 and 6. Here we shall simply mention that the Hpp-wave (3.1) is invariant under the action of the Lie algebra $\mathfrak{g}_A \rtimes (\mathfrak{s}_A \cap \mathfrak{s}_\Theta)$, where $\mathfrak{s}_A \subset \mathfrak{so}(V)$ and $\mathfrak{s}_\Theta \subset \mathfrak{so}(V)$ are the isotropy algebras of A and Θ, respectively, and $\rtimes$ is the semidirect sum. The action of $\mathfrak{s}_A \cap \mathfrak{s}_\Theta$ on $\mathfrak{g}_A$ is induced by restriction from the natural action of $\mathfrak{so}(V)$ on $V \oplus V^* \subset \mathfrak{g}_A$. In the next section we shall show that generic Hpp-wave solutions preserve one half of the supersymmetry.

There are some special cases of Hpp-waves that we can consider. One possibility is to choose $\Theta = 0$. In such case, the spacetime is not necessarily Minkowski. The condition (3.2) only implies that the trace of A vanishes. The associated CW spaces are *Ricci flat* but not isometric to Minkowski space. However if $A = 0$, we recover eleven-dimensional Minkowski space. Therefore the moduli space of Hpp-waves has a point that preserves thirty-two supersymmetries. As we will see presently, it has exactly one other such point.

Another case that we shall focus later is to take

$$\begin{aligned} ds^2 &= 2dx^+dx^- + \sum_{i,j} A_{ij} x^i x^j (dx^-)^2 + \sum_i dx^i dx^i \,, \\ F &= \mu\, dx^- \wedge dx^1 \wedge dx^2 \wedge dx^3 \,, \end{aligned} \tag{3.7}$$

where μ is a constant. In such a case eq. (3.2) implies that

$$\operatorname{tr} A = -\frac{1}{2}\mu^2 \,. \tag{3.8}$$

The isotropy algebra of F is $\mathfrak{so}(3) \oplus \mathfrak{so}(6)$. We shall show in section 4 that for a special choice of A, this solution preserves thirty-two supersymmetries. This is the KG solution of eleven-dimensional supergravity.

Another special case of (3.7) is the solution given in [5]. This solution corresponds to a reducible CW space. To see this, we write the metric given in [5] as

$$ds^2 = -dt^2 + dx_9^2 + 2br^2 d\varphi(dx_9 - dt) + dr^2 + r^2 d\varphi^2 + ds^2(\mathbb{R}^7) \,.$$

Next we change coordinates

$$x^\pm = \frac{1}{\sqrt{2}}(\pm t + x_9) \,, \qquad \tilde\varphi = \varphi + b(x_9 - t) \,.$$

In these new coordinates the metric is written as

$$ds^2 = 2dx^-dx^+ - 2b^2r^2(dx^-)^2 + dr^2 + r^2 d\tilde\varphi^2 + ds^2(\mathbb{R}^7) \,.$$

After periodically identifying $\tilde\varphi$ and changing from polar $(r, \tilde\varphi)$ to rectangular coordinates on $\mathbb{R}^2$, the metric we find is that of a decomposable CW space for which A has two equal non-vanishing eigenvalues $\lambda_1 = \lambda_2 = -2b^2$.

4. Killing spinors

There are two cases to consider. First we shall show that generic Hpp-waves (3.1) preserve one half of the supersymmetry, that is, that they admit sixteen linearly independent Killing spinors. Then we shall prove that there is a special point in the moduli space of these solutions, which is not Minkowski space, where supersymmetry is enhanced and the solution admits thirty-two linearly independent Killing spinors. In particular, we shall show that if

$$
\begin{aligned}
A_{ij} &= \begin{cases} -\frac{1}{9}\mu^2\delta_{ij}\,, & \text{i,j=1,2,3}\,, \\ -\frac{1}{36}\mu^2\delta_{ij}\,, & \text{i,j=4,5,}\ldots\text{,9}\,, \end{cases} \\
\Theta &= \mu dx^1 \wedge dx^2 \wedge dx^3
\end{aligned}
\tag{4.1}
$$

then the solution (3.7) preserves all supersymmetry. This is the KG vacuum solution of eleven-dimensional supergravity theory.

To begin the analysis, the non-vanishing components of the spin connection one-form of the (3.7) solution are

$$
\omega^{+i} = -\omega^{i+} = \sum_j A_{ij} x^j dx^- \,,
$$

where $+i$ are frame indices. In addition, using that $F_{-123} = \mu$ and all other components vanish, the F-dependent piece Ω_M of the Killing spinor equation (3.4) has the following non-zero components[2]

$$
\begin{aligned}
\Omega_- &= -\frac{1}{12}\mu\left(\Gamma_+\Gamma_- + \mathbb{1}\right) I\,, \\
\Omega_i &= \begin{cases} \frac{1}{6}\mu\Gamma_+\Gamma_i I\,, & \text{i=1,2,3}\,, \\ -\frac{1}{12}\mu\Gamma_+\Gamma_i I\,, & \text{i=4,5,}\ldots\text{,9}\,, \end{cases}
\end{aligned}
\tag{4.2}
$$

where $I = \Gamma_{123}$ obeys $I^2 = -\mathbb{1}$. It follows from $\Gamma_+^2 = 0$ that

$$
\Omega_i \Omega_j = 0\,, \qquad \text{for all } i,j = 1,2,\ldots,9\,. \tag{4.3}
$$

To solve the Killing spinor equations for the (generic) solution (3.1), we impose the condition

$$
\Gamma_+ \varepsilon = 0\,. \tag{4.4}
$$

In this case, the Killing spinor equations reduce to

$$
\partial_- \varepsilon = \frac{1}{24}\Theta_{ijk}\Gamma^{ijk}\varepsilon\,,
$$

where $\varepsilon = \varepsilon(x^-)$ is only a function of x^-. This equation can be solved and the Killing spinors are

$$
\varepsilon = \exp\left(\frac{1}{24}x^- \Theta_{ijk}\Gamma^{ijk}\right)\psi_+\,, \tag{4.5}
$$

for some constant spinor ψ_+ such that $\Gamma_+\psi_+ = 0$. It is clear from this that the generic solution (3.1) preserves one half of the supersymmetry.

[2]Throughout this section, the indices on the Γ-matrices are frame indices.

In particular for the solution (3.7), the Killing spinor equations are

$$\partial_- \varepsilon + \frac{\mu}{4} I \varepsilon = 0 \,, \tag{4.6}$$

and the Killing spinors are given as

$$\varepsilon = \left(\cos\left(\frac{\mu}{4} x^-\right) \mathbb{1} - \sin\left(\frac{\mu}{4} x^-\right) I \right) \psi_+ \,,$$

where ψ_+ is again a constant spinor satisfying $\Gamma_+ \psi_+ = 0$.

Next we turn to find the special point in the moduli space of (3.7) solutions that exhibits enhancement of supersymmetry. Since $\nabla_+ = \partial_+$ and $\Omega_+ = 0$, we see that the Killing spinor ε is independent of x^+. Similarly from

$$\partial_i \varepsilon = \Omega_i \varepsilon \tag{4.7}$$

and eq. (4.3), we see that $\partial_i \partial_j \varepsilon = 0$, whence ε is at most linear in x^i. Let us write it as

$$\varepsilon = \chi + \sum_i x^i \varepsilon_i \,,$$

where the spinors χ and ε_i are only functions of x^-. From eq. (4.7) we see that $\varepsilon_i = \Omega_i \chi$, whence any Killing spinor ε takes the form

$$\varepsilon = \left(\mathbb{1} + \sum_i x^i \Omega_i \right) \chi \,, \tag{4.8}$$

where the spinor χ only depends on x^-. The dependence on x^- is fixed from the one remaining equation

$$\partial_- \varepsilon = -\frac{1}{2} \sum_{i,j} A_{ij} x^j \Gamma_+ \Gamma_i \varepsilon - \frac{1}{12} \mu \left(\Gamma_+ \Gamma_- + \mathbb{1} \right) I \varepsilon \,,$$

which will also imply an integrability condition fixing A.

Inserting the above expression (4.8) for ε into this equation and after a little bit of algebra (using repeatedly that $\Gamma_+^2 = 0$), we find

$$\begin{aligned} \frac{d}{dx^-} \chi = & -\frac{1}{12} \mu I \left(\mathbb{1} + \Gamma_+ \Gamma_- \right) \chi + \\ & + \sum_i x^i \left(-\frac{1}{2} \sum_j A_{ij} \Gamma_+ \Gamma_j + \frac{1}{12} \mu \Omega_i I - \frac{1}{4} \mu I \Omega_i \right) \chi \,. \end{aligned} \tag{4.9}$$

Because χ is independent of x^i, the terms in parenthesis in the right-hand side of the equation must vanish separately. This will fix A. The remaining equation is a first-order linear ordinary differential equation with constant coefficients, which has a unique solution for each initial value. Since the initial value is an arbitrary spinor, we see that the dimension of the space of Killing spinors is 32 and hence the solution will be maximally supersymmetric.

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To fix A, notice that

$$\Omega_i I = \begin{cases} -\frac{1}{6}\mu\Gamma_+\Gamma_i\,, & \text{i=1,2,3}\,,\\ \frac{1}{12}\mu\Gamma_+\Gamma_i\,, & \text{i=4,5,}\ldots\text{,9}\end{cases}$$

and

$$I\Omega_i = \begin{cases} \frac{1}{6}\mu\Gamma_+\Gamma_i\,, & \text{i=1,2,3}\,,\\ \frac{1}{12}\mu\Gamma_+\Gamma_i\,, & \text{i=4,5,}\ldots\text{,9}\end{cases}$$

whence the x^i-dependent terms in (4.9) vanish provided that

$$\sum_j A_{ij}\Gamma_j = \begin{cases} -\frac{1}{9}\mu^2\Gamma_i\,, & \text{i=1,2,3}\,,\\ -\frac{1}{36}\mu^2\Gamma_i\,, & \text{i=4,5,}\ldots\text{,9}\,,\end{cases}$$

whence A is diagonal with eigenvalues $\lambda_i = -\frac{1}{9}\mu^2$ for $i = 1, 2, 3$ and $\lambda_i = -\frac{1}{36}\mu^2$ for $i = 4, 5, \ldots, 9$. In particular it is non degenerate, whence the metric (2.1) is indecomposable. As a final check, notice that $\operatorname{tr} A = -\frac{1}{2}\mu^2$ as required from the equations of motion. We conclude therefore that the solution (3.7) with A given in (4.1) is a maximally supersymmetric solution of eleven-dimensional supergravity. Notice that all non-zero values of μ are isometric (simply rescale x^-), whereas $\mu = 0$ corresponds to eleven-dimensional Minkowski spacetime.

One can give a more explicit expression for the Killing spinors. Decompose the spinor χ as $\chi_+ + \chi_-$, where $\Gamma_\pm\chi_\pm = 0$. Since I preserves $\ker\Gamma_\pm$, the first-order equation for χ breaks up into two equations:

$$\frac{d}{dx^-}\chi_+ = -\frac{1}{4}\mu I\chi_+\,, \qquad \text{and} \qquad \frac{d}{dx^-}\chi_- = -\frac{1}{12}\mu I\chi_-\,,$$

which can be solved via matrix exponentials in terms of constant spinors $\psi_\pm \in \ker\Gamma_\pm$:

$$\begin{aligned}\chi_+ &= \exp\left(-\frac{1}{4}\mu x^- I\right)\psi_+ = \left(\cos\left(\frac{1}{4}\mu x^-\right)\mathbb{1} - \sin\left(\frac{1}{4}\mu x^-\right) I\right)\psi_+\,,\\ \chi_- &= \exp\left(-\frac{1}{12}\mu x^- I\right)\psi_- = \left(\cos\left(\frac{1}{12}\mu x^-\right)\mathbb{1} - \sin\left(\frac{1}{12}\mu x^-\right) I\right)\psi_-\,.\end{aligned}$$

Finally, we use eq. (4.8) to arrive at the following expression for ε in terms of the arbitrary constant spinors $\psi_\pm$:

$$\begin{aligned}\varepsilon = &\left(\cos\left(\frac{1}{4}\mu x^-\right)\mathbb{1} - \sin\left(\frac{1}{4}\mu x^-\right) I\right)\psi_+ +\\ &+\left(\cos\left(\frac{1}{12}\mu x^-\right)\mathbb{1} - \sin\left(\frac{1}{12}\mu x^-\right) I\right)\psi_- -\\ &-\frac{1}{6}\mu\left(\sum_{i\leq 3} x^i\Gamma_i - \frac{1}{2}\sum_{i\geq 4} x^i\Gamma_i\right)\left(\sin\left(\frac{1}{12}\mu x^-\right)\mathbb{1} - \cos\left(\frac{1}{12}\mu x^-\right) I\right)\Gamma_+\psi_-\,.\end{aligned} \tag{4.10}$$

Observe that the Killing spinors depend trigonometrically on x^-. These expressions agree (up to notation) with those found in [6] for the KG solution.

5. Isometries

We shall begin by investigating the isometries of a generic CW space M_A and then we shall specialise to those of the KG solution. Since $M_A = G_A/K_A$ is the space of left cosets of K_A in G_A, it admits a natural action of G_A. By construction, the metric on M_A is invariant under this action and hence the associated transformations are isometries. This means that there are 20 linearly independent Killing vector fields ξ_X each associated with an element X of the Lie algebra $\mathfrak{g}_A$ of G_A. The map $X \mapsto \xi_X$ is an isomorphism of Lie algebras, where the Lie bracket of ξ_X is the Lie bracket of vector fields on M_A. To express these Killing vector fields in terms of the coordinates $\{x^\pm, x^i\}$, we shall use the local coset representative σ given by (2.4) and act on it with G_A from the left. The resulting right-invariant vector fields project to Killing vectors on M_A relative to the G_A-invariant metric. To simplify the expressions for the Killing vector fields, we will assume that the coordinates $\{x^i\}$ have been chosen in such a way that A is diagonalised: $A_{ij} = \lambda_i \delta_{ij}$.

Let $X \in \mathfrak{g}_A$ and let $t \mapsto \exp tX$ be a one-parameter subgroup of G_A with tangent vector X at the identity. Acting with this one-parameter subgroup on $\sigma(x)$, we obtain

$$\exp(tX)\,\sigma(x) = \sigma\left(\exp(tX)\cdot x\right) k(t,x)\,,$$

for some $k(t,x) \in K_A$. Notice that $k(0,x)$ is the identity for all x, hence differentiating $k(t,x)$ with respect to t at $t=0$ we obtain an element (Y, say) of $\mathfrak{k}_A$. Differentiating the previous equation with respect to t at $t=0$ we therefore obtain (abusing notation slightly)

$$X\sigma(x) = \xi_X + \sigma(x) Y\,,$$

where $Y \in \mathfrak{k}_A$ and where ξ_X is the local coordinate expression for the Killing vector corresponding to X. In other words,

$$\xi_X = X\sigma(x) \pmod{\sigma(x)\mathfrak{k}_A}\,.$$

To express ξ_X in terms of the coordinate vectors $\{\partial_\pm, \partial_i\}$, we need to recognise these in G_A. Recalling that the action on coordinates is inverse to that on points, the vector fields ∂_+, ∂_- and ∂_i at the point with coordinates $\{x^+, x^-, x^j\}$ are the tangent vectors, respectively, to the following curves on M_A.

$$t \longmapsto (x^+ - t, x^-, x^j)\,, \qquad t \longmapsto (x^+, x^- - t, x^j)\,, \qquad t \longmapsto (x^+, x^-, x^j - t\delta_{ij})\,.$$

We can think of these as curves on G_A by composing them with σ. Then their tangent vectors at the point $\sigma(x)$ become (with a slight abuse of notation):

$$-e_+\sigma(x)\,, \qquad -e_-\sigma(x)\,, \qquad -\sigma(x)e_i\,,$$

which are the expressions in G_A for (the push-forwards via σ of) ∂_+, ∂_- and ∂_i, respectively. Using the above computations, we find that the Killing vector fields associated with the action of G_A on the coset M_A are the following:

$$\begin{aligned} \xi_{e_+} &= -\partial_+\,, \qquad \xi_{e_-} = -\partial_-\,, \\ \xi_{e_i} &= -C_i(x^-)\partial_i + S_i(x^-)x^i\lambda_i\partial_+\,, \\ \xi_{e_i^*} &= \lambda_i S_i(x^-)\partial_i - C_i(x^-)x^i\lambda_i\partial_+\,, \end{aligned}$$

(there is no sum over i), where the functions $C_i(x^-)$ and $S_i(x^-)$ are given by

$$C_i(x^-) = \begin{cases} 1\,, & \text{if } \lambda_i = 0\,, \\ \cosh\left(\sqrt{\lambda_i}\,x^-\right), & \text{if } \lambda_i > 0\,, \\ \cos\left(\sqrt{-\lambda_i}\,x^-\right), & \text{if } \lambda_i < 0\,, \end{cases}$$

and

$$S_i(x^-) = \begin{cases} x^-\,, & \text{if } \lambda_i = 0\,, \\ \frac{\sinh\left(\sqrt{\lambda_i}\,x^-\right)}{\sqrt{\lambda_i}}\,, & \text{if } \lambda_i > 0\,, \\ \frac{\sin\left(\sqrt{-\lambda_i}\,x^-\right)}{\sqrt{-\lambda_i}} & \text{if } \lambda_i < 0\,, \end{cases}$$

respectively. One can check that the Killing vector fields obtained above do indeed form a representation of $\mathfrak{g}_A$. We note that the supergravity solution of section 3 associated with the CW space M_A is also invariant under the above isometries; that is, the above isometries leave invariant the four-form field-strength F as well.

For generic A these are all the Killing vectors of the metric (2.1), but if A is invariant under some subalgebra $\mathfrak{s}_A \subset \mathfrak{so}(V)$, then there are extra Killing vectors corresponding to $\mathfrak{s}_A$. Typically this occurs whenever two or more eigenvalues of A coincide. The subalgebra $\mathfrak{s}_A$ is generated by the rotations that leave invariant the eigenspace corresponding to these eigenvalues. The Lie algebra $\mathfrak{s}_A$ acts naturally on $\mathfrak{g}_A$ by restricting the action of $\mathfrak{so}(V)$ on $V \oplus V^* \subset \mathfrak{g}_A$. Since $\mathfrak{s}_A$ leaves A invariant, it preserves the Lie bracket on $\mathfrak{g}_A$ and hence we can define the semi-direct product $\mathfrak{g}_A \rtimes \mathfrak{s}_A$. It can be shown that this is the isometry algebra of M_A. The unenlightening proof (which we omit) involves solving Killing's equation explicitly.

The symmetries of the Hpp-waves solution (3.1) leave both the eleven-dimensional metric and four-form field strength invariant. Adapting the explanation we have presented above for M_A it is easy to see that the symmetry group of Hpp-waves is $\mathfrak{g}_A \rtimes (\mathfrak{s}_A \cap \mathfrak{s}_\Theta)$, where $\mathfrak{s}_\Theta$ is the subalgebra of $\mathfrak{so}(V)$ that leaves the three-form Θ invariant. For example, if $A_{ij} = \delta_{ij}$, then the algebra of isometries of the CW space M_I is $\mathfrak{g}_A \rtimes \mathfrak{so}(9)$. For the associated Hpp-wave (3.7), however, the symmetry subalgebra is $\mathfrak{g}_A \rtimes (\mathfrak{so}(3) \oplus \mathfrak{so}(6))$.

Now we shall investigate the symmetries of the maximally supersymmetric KG solution, which was originally computed in [6]. The eigenvalues of A given by (4.1) are negative, hence just like the Killing spinors, the Killing vectors coming from the

G_A-action depend trigonometrically on x^-. Apart from the symmetries associated with $\mathfrak{g}_A$, this solution also has an additional $\mathfrak{s}_A \cong \mathfrak{so}(3) \oplus \mathfrak{so}(6)$ invariance because A has two distinct eigenvalues with three- and six-dimensional eigenspaces, respectively. The Killing vectors of the KG background are the following:

$$\begin{aligned}
\xi_{e_+} &= -\partial_+ \,, \qquad \xi_{e_-} = -\partial_- \,, \\
\xi_{e_i} &= -\cos\left(\frac{\mu x^-}{3}\right)\partial_i - \sin\left(\frac{\mu x^-}{3}\right)\frac{\mu x^i}{3}\partial_+ \,, \qquad i = 1,2,3\,, \\
\xi_{e_i^*} &= -\sin\left(\frac{\mu x^-}{3}\right)\frac{\mu}{3}\partial_i + \cos\left(\frac{\mu x^-}{3}\right)\frac{\mu^2 x^i}{9}\partial_+ \,, \qquad i = 1,2,3\,, \\
\xi_{e_i} &= -\cos\left(\frac{\mu x^-}{6}\right)\partial_i - \sin\left(\frac{\mu x^-}{6}\right)\frac{\mu x^i}{6}\partial_+ \,, \qquad i = 4,5,6,7,8,9\,, \\
\xi_{e_i^*} &= -\sin\left(\frac{\mu x^-}{6}\right)\frac{\mu}{6}\partial_i + \cos\left(\frac{\mu x^-}{6}\right)\frac{\mu^2 x^i}{36}\partial_+ \,, \qquad i = 4,5,6,7,8,9\,, \\
\xi_{M_{ij}} &= x^i\partial_j - x^j\partial_i \,, \qquad i,j = 1,2,3\,, \\
\xi_{M_{ij}} &= x^i\partial_j - x^j\partial_i \,, \qquad i,j = 4,5,6,7,8,9\,,
\end{aligned} \tag{5.1}$$

where $\{M_{ij}; i,j = 1,2,3\}$ and $\{M_{ij}; i,j = 4,5,6,7,8,9\}$ are generators of $\mathfrak{so}(3)$ and $\mathfrak{so}(6)$, respectively.

The bosonic generators of the symmetry algebra of the KG solution is $\mathfrak{g}_A \rtimes (\mathfrak{so}(3) \oplus \mathfrak{so}(6))$ has dimension 38, which (intriguingly) is of the same dimension as the isometry algebras of the other non-trivial maximally supersymmetric solutions: $AdS_4 \times S^7$ and $AdS_7 \times S^4$.

6. The symmetry superalgebra

In addition to the bosonic symmetries generated by Killing vector fields, solutions of supergravity theories preserving some supersymmetry are also invariant under fermionic symmetries generated by shifts along their Killing spinors. Combining the bosonic and fermionic symmetries, such backgrounds are invariant under the action of a Lie superalgebra. The technique employed in this section to construct this Lie superalgebra has been described in the context of backgrounds of the form $AdS_p \times X$ in [15, 16].[3] In this section we apply it to the M-theory Hpp-waves of (3.1). We shall first find the symmetry superalgebra of the KG solution and then we shall give the symmetry superalgebra of a generic Hpp-wave.

The Lie superalgebra in question will be denoted $\mathfrak{S}$, and it splits into even and odd subspaces $\mathfrak{S} = \mathfrak{S}_0 \oplus \mathfrak{S}_1$. The odd subspace is spanned by the Killing spinors

[3]These papers review the application of this technique in [17] and [18] respectively; however we would be very surprised if this technique had not already appeared either implicitly or explicitly in the early supergravity literature.

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and the even subspace by those Killing vectors also preserving F. In order to define the structure of a Lie superalgebra we need to construct linear maps

$$\begin{aligned}[\cdot,\cdot]:&\ \mathfrak{S}_0 \times \mathfrak{S}_0 \longrightarrow \mathfrak{S}_0\,,\\ [\cdot,\cdot]:&\ \mathfrak{S}_0 \times \mathfrak{S}_1 \longrightarrow \mathfrak{S}_1\,,\\ \{\cdot,\cdot\}:&\ \mathfrak{S}_1 \times \mathfrak{S}_1 \longrightarrow \mathfrak{S}_0\,,\end{aligned}$$

subject to the Jacobi identities. The first map is simply the Lie bracket of vector fields and it clearly satisfies the Jacobi identity. The second map is the action of Killing vectors on Killing spinors, making the space of Killing spinors into a linear representation of the isometry subalgebra. As in [15, 16], this is achieved by the spinorial Lie derivative. If X is a Killing vector, then one can define a Lie derivative L_X on a spinor ψ as

$$L_X\psi = X^M\nabla_M\psi + \frac{1}{4}\nabla_{[M}X_{N]}\Gamma^{MN}\psi\,.$$

As it has been explained in [15], this has the following properties:

(1) If f is any smooth function and ψ is any spinor, then

$$L_X(f\psi) = (Xf)\psi + f L_X\psi\,;$$

(2) If X is a Killing vector field, Y is *any* vector field, and ψ any spinor, then

$$L_X(Y\cdot\psi) = [X,Y]\cdot\psi + Y\cdot L_X\psi\,,$$

where $\cdot$ denotes the Clifford action of vectors on spinors; and

(3) If X, Y are Killing vector fields and ψ is a spinor,

$$L_XL_Y\psi - L_YL_X\psi = L_{[X,Y]}\psi\,.$$

Notice that the spinorial Lie derivative preserves the Spin-invariant inner product on the space of spinors.

If X is a Killing vector which in addition preserves the four-form F, then it is easy to show that L_X preserves the space of Killing spinors $\mathfrak{S}_1$ of a eleven-dimensional background. Therefore $\mathfrak{S}_1$ becomes a representation of the Lie algebra $\mathfrak{S}_0$: the Lie algebra spanned by Killing vectors which preserve F.

Finally, the last map $\{\cdot,\cdot\}$ in the structure of the Lie superalgebra $\mathfrak{S}$ is simply the squaring of Killing spinors. Indeed, it is easy to show that if ε_i, $i = 1, 2$ are Killing spinors, then the vector field with components $\bar{\varepsilon}_1\Gamma^M\varepsilon_2$ is a Killing vector. For the background in question, these Killing vectors also preserve F. The second property of the Lie derivative guarantees that this operation is equivariant under the action of isometries, which is one component of the Jacobi identity for the Lie superalgebra $\mathfrak{S}$.

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We will now explicitly exhibit the symmetry superalgebra. The commutators of the bosonic part of the superalgebra are

$$
\begin{aligned}
&[e_-,e_i]=e_i^*\,, \qquad [e_-,e_i^*]=-\frac{\mu^2}{9}e_i\,, \qquad (i\leq 3)\,, \qquad [e_-,e_i^*]=-\frac{\mu^2}{36}e_i\,, \qquad (i\geq 4)\,,\\
&[e_i^*,e_j]=-\frac{\mu^2}{9}e_+\,, \qquad (i,j\leq 3)\,, \qquad [e_i^*,e_j]=-\frac{\mu^2}{36}e_+\,, \qquad (i,j\geq 4)\,,\\
&[M_{ij},e_k]=-\delta_{ik}e_j+\delta_{jk}e_i\,, \qquad \text{for} \qquad (i,j,k\leq 3) \ \text{ and } \ (i,j,k\geq 4)\,,\\
&[M_{ij},e_k^*]=-\delta_{ik}e_j^*+\delta_{jk}e_i^*\,, \qquad \text{for} \qquad (i,j,k\leq 3) \ \text{ and } \ (i,j,k\geq 4)\,.
\end{aligned} \tag{6.1}
$$

To continue with the computation, we introduce odd generators $Q_\pm$ which generate shifts along the the constant spinor $\psi_\pm$ parameterising the Killing spinors in (4.10). As we have argued, the spinorial Lie derivative preserves the space of Killing spinors. Let ξ be a Killing vector field. Acting on a generic Killing spinor $\varepsilon(\psi_+,\psi_-)$, L_χ will give a Killing spinor with different parameters $\varepsilon(S_\xi^-\psi_+,S_\xi^+\psi_-)$. This defines an action of the Lie algebra of isometries on the space of Killing spinors, whose structure constants are given by the constant matrices S_ξ^- and S_ξ^+. An explicit computation reveals that

$$
\begin{aligned}
&[e_+,Q_\pm]=0\,, \qquad [e_-,Q_+]=-\frac{\mu}{4}IQ_+\,, \qquad [e_-,Q_-]=-\frac{\mu}{12}IQ_-\,,\\
&[e_i,Q_+]=-\frac{\mu}{6}I\Gamma_i\Gamma_+Q_-\,, \qquad (i\leq 3)\,, \qquad [e_i,Q_+]=-\frac{\mu}{12}I\Gamma_i\Gamma_+Q_-\,, \qquad (i\geq 4)\,,\\
&[e_i^*,Q_+]=-\frac{\mu^2}{18}\Gamma_i\Gamma_+Q_-\,, \qquad (i\leq 3)\,, \qquad [e_i^*,Q_+]=-\frac{\mu^2}{72}\Gamma_i\Gamma_+Q_-\,, \qquad (i\geq 4)\,,\\
&[M_{ij},Q_\pm]=\frac{1}{2}\Gamma_{ij}Q_\pm\,, \qquad (i,j\leq 3) \ \text{ and } \ (i,j\geq 4)\,.
\end{aligned} \tag{6.2}
$$

Using the Clifford algebra, it is easy to check that this forms a representation of $\mathfrak{g}_A \rtimes \mathfrak{s}_A$.

Finally we compute the bracket $\{\cdot,\cdot\}$ of odd generators of the symmetry superalgebra. For this let $\varepsilon_1=\varepsilon(\psi_+,\psi_-)$ and $\varepsilon_2=\varepsilon(\rho_+,\rho_-)$ be Killing spinors associated with constant spinors (ψ_+,ψ_-) and (ρ_+,ρ_-) as in (4.10), respectively. Then the vector $V=\bar{\varepsilon}_1\Gamma^M\varepsilon_2\partial_M$ is a linear combination of Killing vectors with constant coefficients:

$$
\begin{aligned}
V \;=\; &-(\bar{\psi}_-\Gamma_+\rho_-)\,\xi_{e_-}-(\bar{\psi}_+\Gamma_-\rho_+)\,\xi_{e_+}+\frac{\mu}{6}\sum_{i,j\leq 3}(\bar{\psi}_-\Gamma_+I\Gamma_{ij}\rho_-)\,\xi_{M_{ij}}-\\
&-\frac{\mu}{12}\sum_{i,j\geq 4}(\bar{\psi}_-\Gamma_+I\Gamma_{ij}\rho_-)\,\xi_{M_{ij}}-\sum_{i\leq 3}\left(\bar{\psi}_+\Gamma_i\rho_-+\bar{\psi}_-\Gamma_i\rho_+\right)\xi_{e_i}+\\
&+\frac{3}{\mu}\sum_{i\leq 3}\left(\bar{\psi}_+I\Gamma_i\rho_-+\bar{\psi}_-I\Gamma_i\rho_+\right)\xi_{e_i^*}-\sum_{i\geq 4}\left(\bar{\psi}_+\Gamma_i\rho_-+\bar{\psi}_-\Gamma_i\rho_+\right)\xi_{e_i}+\\
&+\frac{6}{\mu}\sum_{i\geq 4}\left(\bar{\psi}_+I\Gamma_i\rho_--\bar{\psi}_-I\Gamma_i\rho_+\right)\xi_{e_i^*}\,,
\end{aligned} \tag{6.3}
$$

where M_{ij} are the generators of the isotropy algebra $\mathfrak{s}_A$ of A. From this one can read the anticommutators of the odd generators of the superalgebra as

$$
\begin{aligned}
\{Q_+, Q_+\} &= -\Gamma_- C^{-1} e_+ \,, \\
\{Q_+, Q_-\} &= -\sum_{i=1}^{9} \Gamma^i C^{-1} e_i + \frac{3}{\mu} \sum_{i\leq 3} I\Gamma^i C^{-1} e_i^* + \frac{6}{\mu} \sum_{i\geq 4} I\Gamma^i C^{-1} e_i^* \,, \\
\{Q_-, Q_-\} &= -\Gamma_+ C^{-1} e_- + \frac{\mu}{6} \sum_{i,j\leq 3} \Gamma_+ I\Gamma^{ij} C^{-1} M_{ij} - \\
&\quad - \frac{\mu}{12} \sum_{i,j\geq 4} \Gamma_+ I\Gamma^{ij} C^{-1} M_{ij} \,.
\end{aligned} \tag{6.4}
$$

To summarise the (anti)commutation relations of the symmetry superalgebra of the KG solution are given by the equations (6.1), (6.2) and (6.4).

The symmetry superalgebra of an M-theory Hpp-wave solution (3.1) based on a generic CW space is the following. The commutators of the bosonic generators are given in (2.2). This solution preserves only half of the supersymmetry and since the Killing spinor ε satisfies $\Gamma_+\varepsilon = 0$, the associated superalgebra contains only Q_+ generators. It follows that the remaining non-vanishing commutators and anticommutators are

$$
\begin{aligned}
[e_-, Q_+] &= -\frac{1}{24}\Theta_{ijk}\Gamma^{ijk} Q_+ \,, \qquad [M_{ij}, Q_+] = \frac{1}{2}\Gamma_{ij} Q_+ \,, \\
\{Q_+, Q_+\} &= -\Gamma_- C^{-1} e_+ \,,
\end{aligned} \tag{6.5}
$$

where M_{ij} are generators of $\mathfrak{s}_A \cap \mathfrak{s}_\Theta$. The symmetry enhancement at the special point is dramatically illustrated by comparing the symmetry superalgebra of the generic Hpp-wave, given by (2.2) and (6.5), with that of the KG solution, given by eqs. (6.1), (6.2) and (6.4).

7. H-branes

Reducing the M-theory Hpp-wave found above and using U-duality, we can construct Hp-brane solutions in all type-II supergravities. The reduction and U-duality for the metric and form-field strengths are straightforward to perform. We should only ensure that at every stage we choose a CW space which is invariant under the direction that the reduction and U-duality are performed. In particular, this implies that Hp-branes for $p > 0$ are associated with decomposable CW spaces.

The preservation of supersymmetry under reduction and U-duality is not obvious. This is because the Killing spinors depend nontrivially on most of the eleven-dimensional coordinates and in particular x^-. We shall come to this point later.

First we shall reduce the M-theory Hpp-wave to IIA along the direction $y = \frac{1}{\sqrt{2}}(x^+ + x^-)$, where we have chosen $x^\pm = (\pm t + x^{10})/\sqrt{2}$. We shall focus in the reduction of the Hpp-wave given in (3.7). This is because the four-form field strength vanishes along the direction x^i for $i > 3$ and the Killing spinors depend trigonometrically on x^-. Using the Kaluza-Klein Ansatz leading to the string frame in IIA theory, we find the following H0-brane solution:

$$\begin{aligned}
ds^2 &= -\Lambda^{-1/2}dt^2 + \Lambda^{1/2}ds^2(\mathbb{R}^9)\,, \\
F_4 &= -\frac{\mu}{\sqrt{2}}dt \wedge dx^1 \wedge dx^2 \wedge dx^3\,, \\
H_3 &= \frac{\mu}{\sqrt{2}}dx^1 \wedge dx^2 \wedge dx^3\,, \\
F_2 &= -dt \wedge d\Lambda^{-1}\,, \\
e^{2\phi} &= \Lambda^{3/2}\,,
\end{aligned} \tag{7.1}$$

where $\Lambda = 1 + \frac{1}{2}A_{ij}x^ix^j$, F_2, F_4 are the RR field strengths and H_3 is the NSNS three form field-strength. Observe that near the origin $x^i = 0$, the metric of the H0-brane approaches Minkowski space, the dilaton is constant, the two-form field strength vanishes, and both H_3 and F_4 are constant. Away from the origin, the dilaton, and so the string coupling, increases or decreases depending on whether along those directions A is positive- or negative-definite. Along the directions that A is negative-definite the dilaton will eventually become complex signalling an instability in the theory. Despite the complicated behaviour of the solution in ten dimensions, the associated Hpp-wave solution in eleven dimensions is smooth and preserves at least one half of the supersymmetry. For example, the Hpp-wave associated with the KG background has negative-definite A and so the dilaton becomes complex for all x such that

$$\frac{\mu^2}{9}\left(\sum_{i\leq 3}(x^i)^2 + \frac{1}{4}\sum_{i\geq 4}(x^i)^2\right) > 1\,.$$

The associated Hpp-wave preserves all supersymmetry.

To construct other Hp-branes for $p > 0$, one performs T-duality along the directions x^{3+q} for $1 \leq q \leq p$ on the H0-brane (7.1). We now take $A_{ij}x^ix^j$ to be independent from x^{3+q} in which case the CW space becomes degenerate. For this one can use the T-duality rules of [19]. Although the application of these rules is straightforward, the expressions for the Hp-branes are rather involved mainly due to the fact that the H0-brane has non-vanishing form field-strengths from both the NSNS and RR sectors. One can easily see that the metric and the dilaton for an Hp-brane are given by

$$\begin{aligned}
ds^2 &= \Lambda^{-1/2}\left(-dt^2 + \sum_{q=1}^{p}(dx^{3+q})^2\right) + \\
&\quad + \Lambda^{1/2}\left((dx^1)^2 + (dx^2)^2 + (dx^3)^2 + \sum_{i>p+3}^{9}(dx^i)^2\right), \\
e^{2\phi} &= \Lambda^{\frac{3-p}{2}}\,.
\end{aligned}$$

8. Supersymmetry and H0-branes

To investigate whether supersymmetry is preserved in the construction of H0-brane from the Hpp-wave above, we observe that the metric, the four-form and Killing spinors are independent of x^+ and depend trigonometrically on x^-. For the case of M-theory Hpp-waves (3.7) that preserve one half of the supersymmetry, we can identify $x^+ \mapsto x^+ + 2\pi m\ell$ for any ℓ and $x^- \mapsto x^- + \frac{4\pi}{\mu}n$, for some $m, n \in \mathbb{Z}$. For even n we must take the trivial spin structure on this torus, whereas for n odd we must take the non-trivial spin structure on the x^- circle, as the Killing spinors change sign. To reduce the metric and the four-form field strength of the Hpp-wave as above, we compactify along the direction x^{10} by setting $x^{10} \mapsto x^{10} + 2\pi nR$ for some $n \in \mathbb{Z}$ and $R \in \mathbb{R}$; $x^{10} = \frac{1}{\sqrt{2}}(x^+ + x^-)$. Such identification is consistent with supersymmetry in eleven dimensions provided that either $R = \sqrt{2}\ell = 2\sqrt{2}/\mu$ or $R = \sqrt{2}\ell = 4\sqrt{2}/\mu$ and an appropriate spin structure is chosen.

The above relations between the radius of compactification and the periodicity of Killing spinors, although necessary, are not sufficient for the reduced solution to preserve some of the supersymmetry of IIA supergravity. In fact, additional conditions are required for this to be the case. One such local condition is that the Lie derivative of the Killing spinors should vanish along the Killing vector field generated by translations along the compact direction y. This is similar to the condition required for solutions to preserve supersymmetry after T-duality in the context of toric hyperkähler manifolds [10]; T-duality in the context of supergravity can be seen as hidden symmetry of the reduced theory.

For the case of the Hpp-wave, we can immediately see using (4.5) or (4.6) that

$$L_\xi \varepsilon \neq 0 \,,$$

where ε is the Killing spinor and $\xi\partial_y$ is the vector field tangent to the compact direction; unless $\mu = 0$ and the Hpp-wave solution becomes Minkowski space. Therefore we conclude that the H0-brane does not preserve any supersymmetry of IIA supergravity. This can be verified directly: for example, one sees that the dilatino equation is not satisfied. This is another example of reduction breaking the supersymmetry of a solution in addition to those found in [7, 8, 9, 11].

The above conditions for the supersymmetries preserved by a solution after reduction in the context of supergravity can be stated in terms of the (anti)commutation relations of the symmetry superalgebra of the original solution. To see this, recall that the commutators of the bosonic and fermionic generators of the symmetry superalgebra of a solution are computed by evaluating the Lie derivative of the Killing spinors along the Killing vectors associated with the bosonic generators. It is clear now that the unbroken supersymmetries of the reduced solution are those fermionic symmetry generators of the original solution that *commute* with the bosonic generator that is associated with translations along the compact direction. In the case of the Hpp-wave superalgebra (6.5), since e_- and Q_+ do not commute, the H0-brane does not preserve any supersymmetry.

A similar analysis can be done for the case of Hpp-waves that preserve all thirty-two supersymmetries in eleven dimensions. The only difference is that the identification in this case is $x^+ \mapsto x^+ + 2\pi m\ell$ for any ℓ and $x^- \mapsto x^- + \frac{12\pi}{\mu}n$, for some $m, n \in \mathbb{Z}$. For even n we must take the trivial spin structure on this torus, whereas for n odd we must take the non-trivial spin structure on the x^- circle. Note that if instead we identify $x^+ \mapsto x^+ + 2\pi m\ell$ for any ℓ, $x^- \mapsto x^- + \frac{4\pi}{\mu}n$, for some $m, n \in \mathbb{Z}$ and take an appropriate spin structure, then only sixteen supersymmetries are preserved in eleven dimensions.

The above analysis of supersymmetry suggests that if one does string perturbation theory in the H0-brane background, the resulting spectrum will not be supersymmetric and it will most likely contain tachyonic modes. However if additional possibly non-perturbative states are included in the spectrum, then supersymmetry is restored in the theory.

9. HD-branes and HNS-branes

The solutions of M-theory that we have described in section 3 are suitable for describing the asymptotic region of D0-branes in constant four-form field-strength. Since a constant four-form field-strength does not vanish at infinity, such D0-brane solutions do not approach Minkowski spacetime at infinity. Instead, as we shall see, they approach the solutions we have found in section 3. From the perspective of M-theory, the supergravity description of D0-branes is as pp-waves. It is then easy to see that the solution that describes D0-branes in constant four-form field strength is

$$\begin{aligned} ds^2 &= 2dx^+dx^- + K(dx^-)^2 + \sum_i dx^i dx^i \,, \\ F &= \mu\, dx^- \wedge dx^1 \wedge dx^2 \wedge dx^3 \,, \end{aligned} \tag{9.1}$$

where μ is a constant and the field equations imply that

$$\delta^{ij}\partial_i\partial_j K = -\mu^2 \,. \tag{9.2}$$

A solution is

$$K = A_{ij}x^i x^j + \sum_{A=1}^{N} \frac{P_A}{|x - x_A|^7} \,,$$

where x_A are the positions of the D0-branes and P_A are their tensions. It is straightforward to see that the above solution preserves one half of the supersymmetry. The Killing spinors and symmetry superalgebra are those of the solutions of section 3 preserving one half of the supersymmetry. Away from the D0-branes the solution approaches the solutions we have described in section 3. Near the centres, the solution exhibits the same singularity structure as that of a pp-wave.

As in the case of Hp-branes explained in the previous section to compactify along a space like direction to IIA theory, we choose $x^{10} = \frac{1}{\sqrt{2}}(x^+ + x^-)$. In the string frame, the ten-dimensional solution is given by

$$\begin{aligned} ds^2 &= -\Lambda^{-1/2} dt^2 + \Lambda^{1/2} ds^2(\mathbb{R}^9)\,, \\ F_4 &= -\frac{\mu}{\sqrt{2}} dt \wedge dx^1 \wedge dx^2 \wedge dx^3\,, \\ H_3 &= \frac{\mu}{\sqrt{2}} dx^1 \wedge dx^2 \wedge dx^3\,, \\ F_2 &= -dt \wedge d\Lambda^{-1}\,, \\ e^{2\phi} &= \Lambda^{3/2}\,, \end{aligned}$$

where

$$\Lambda = 1 + \frac{1}{2} K\,. \tag{9.3}$$

If the matrix A is positive-semidefinite, then the dilaton is large at infinity and the string coupling constant is large. On the other hand if A is not positive-semidefinite, then there are regions of spacetime that the dilaton is not defined; that is, it becomes complex and multivalued. Nevertheless the existence of HD0-brane solutions in supergravity theory suggests that there are sectors in the Hilbert space of string theory in backgrounds with homogeneous fluxes which are associated with D0-branes. As in the case of H0-branes, HD0-branes are not supersymmetric in the context of IIA supergravity.

One can construct HDp-branes for $p \leq 6$ by T-dualising along the directions $i > 3$. The solutions are similar to those presented in the previous section for the associated Hp-branes. The only difference is that the function Λ appearing in the HDp-branes is given in (9.3). Using S-duality one can construct HNS-branes and HM-branes. The singularity structure of the new solutions is different from that of the associated standard brane solution. For example consider the metric of a HNS5-brane in the string frame located at $x = 0$

$$ds^2 = ds^2(\mathbb{R}^{1,5}) + \Lambda ds^2(\mathbb{R}^4)\,,$$

where $\Lambda = 1 + A_{ij} x^i x^j + \frac{P}{\delta_{ij} x^i x^j}$; $i, j = 1, \ldots, 4$. For simplicity, we take $A_{ij} = \delta_{ij}$. The HNS5-brane spacetime is complete with two asymptotic regions. The near horizon geometry $|x| \to 0$ is $\mathbb{R}^{1,6} \times S^3$ as expected. However the asymptotic geometry as $|x| \to \infty$ is not flat as the NS5-brane but $\mathbb{R}^{1,5} \times C(S^3)$, where $C(S^3)$ is a conformally flat cone over S^3.

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10. HF-branes

A large class of flux-branes, for example those associated with certain Melvin solutions, can be constructed by reducing a flat spacetime [2, 3]. This class of solutions leads to string theory models which can be solved exactly [5, 20]. Such backgrounds have been used to investigate tachyon instability in closed string theory [5, 21, 22] and establish a conjecture for the equivalence of type-IIA and type-OA string theory [23, 22]; for more recent progress see [24]. Instead of using the flat spacetime, one can reduce one of our Hpp-wave solutions and in this way obtain homogeneous flux-brane solutions in ten dimensions, or HF-*branes* for short.

Instead of considering all configurations that can be found in this way, we shall focus here in an HF-brane that arises from the superposition of a Hpp-wave with the F7-brane. To do this we consider an Hpp-wave (3.1) for which the isotropy group of A contains an SO(2) subgroup and is invariant under translations along an additional direction, say x^3. In this case, it can always be arranged such that

$$A_{ij}x^i x^j = b\left((x^1)^2 + (x^2)^2\right) + \tilde{A}(x)\,,$$

where $\tilde{A}(x) = A_{mn}x^m x^n$ for $m, n \geq 4$, whence $\partial_3 \tilde{A} = 0$.

We perform the reduction along the orbits of the Killing vector field $\xi = \partial_3 + BR\partial_\varphi$, where $x^1 = r\cos\varphi$, $x^2 = r\sin\varphi$, and where we have periodically identified $x^3 \sim x^3 + 2\pi R$. Next we introduce the coordinate $\tilde{\varphi} = \varphi - BRx^3$ which is constant along the orbits of ξ. Expressing the Hpp-wave solution in terms of $\tilde{\varphi}$ and x^3 coordinates, we perform the Kaluza-Klein reduction along x^3. In particular, we shall consider three different choices for the three-form Θ in F of Hpp-wave (3.1). First we take Θ to be non vanishing along the directions x^1, x^2. In this case, the Hpp-wave solution that we shall reduce to IIA is

$$\begin{aligned}
ds^2 &= 2dx^+dx^- + \left(b\left((x^1)^2 + (x^2)^2\right) + \tilde{A}(x)\right)(dx^-)^2 + (dx^1)^2 + (dx^2)^2 + \sum_{i>2}(dx^i)^2\,, \\
F_4 &= dx^- \wedge dx^1 \wedge dx^2 \wedge \left(\sum_{i>3} u_i dx^i\right),
\end{aligned}$$

where b and u_i are constants and $2b + \mathrm{tr}\,\tilde{A} = -\frac{1}{2}u_m u^m$. Reducing along x^3, we find

$$\begin{aligned}
ds^2 &= K^{1/2}\left[2dx^+dx^- + (br^2 + \tilde{A}(x))(dx^-)^2 + \sum_{i>3}(dx^i)^2 + dr^2\right] + K^{-1/2}r^2 d\tilde{\varphi}^2\,, \\
F_2 &= d\left(\frac{Br^2}{K}\right) \wedge d\tilde{\varphi}\,, \qquad H_3 = -BRr dx^- \wedge dr \wedge \left(\sum_{i>3} u_i dx^i\right), \\
F_4 &= r dx^- \wedge dr \wedge d\tilde{\varphi} \wedge \left(\sum_{i>3} u_i dx^i\right), \qquad e^{\frac{4}{3}\phi} = K\,, \qquad (10.1)
\end{aligned}$$

where $K = 1 + B^2R^2r^2$. It is easy to see if one takes $A = 0$, the ten-dimensional solution becomes the F7-brane. Observe though that the worldvolume of the F7-brane in the presence of an Hpp-wave, is no longer a Minkowski space. This is an example of a brane whose worldvolume is curved [25]. It is also straightforward to show that the integral of F_2 over the plane with coordinates $(r, \tilde{\varphi})$ is finite as expected for an F-brane.

Two more cases arise by choosing Θ in different ways from the case we have investigated above. For example, if F_4 vanishes along all directions x^1, x^2, x^3, then the IIA three-form vanishes and the IIA four-form coincides with that of the Hpp-wave (3.1). Alternatively, one can choose F_4 to vanish along the x^1, x^2 directions but to be non vanishing along the x^3 direction. In this case the IIA four-form vanishes but $H_3 = i_3F_4$. In both the above cases the metric and two-form field strength of the IIA theory are as in (10.1).

Since the Killing spinors of the generic Hpp-wave solution (3.1) do not depend on the coordinates x^1, x^2, x^3 involved in the compactification, the HF-brane that we have found in (10.1) may preserve some of supersymmetry of IIA supergravity. The Lie derivative of the Killing spinors along the compact direction vanishes unlike the case of H0-brane described in section 7. However now there is a global condition depending on whether the Killing spinors are periodic or antiperiodic with respect to the compact coordinate. In the latter case supersymmetry is broken. The analysis is similar to that for the standard flux-branes in [22].

There are many other cases to be considered. For example, one can also reduce along a diagonal direction which involves the spacelike direction of x^- and φ. In such a case, the IIA configuration could be interpreted as a superposition of an H0-brane with an F7-brane. Many more solutions can be found by applying U-duality to the above configurations.

11. Concluding remarks

It is natural to ask whether there are more maximally supersymmetric solutions of eleven-dimensional supergravity. In fact there are not, a result which appears in [1]. Therefore the moduli space of simply-connected maximally supersymmetric solutions of M-theory consists of a real line and an additional point. The coordinate on the real line can be chosen to be the scalar curvature of the four-dimensional subspace of the maximally supersymmetric $AdS_4 \times S^7$ and $AdS_7 \times S^4$ solutions. The point with vanishing scalar curvature can be identified with the Minkowski vacuum. The point corresponds to the KG solution, which does not have free parameters. Since the KG solution has vanishing scalar curvature, it could also be identified with the point of vanishing scalar curvature but in this case the moduli space would not be Hausdorff.

The presence of "unexpected" maximally supersymmetric solutions in eleven-dimensional supergravity raises the question whether similar solutions exist in other supergravities. Due to the large number of symmetries, it is expected that string theory on such backgrounds is solvable or amenable to a CFT/AdS type of conjecture.

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